Test in Mathematics

Instruction

This is the computer-based version of the national exam test in mathematics.

The test consists of 40 problems.

The answers for the problems 31-40 must be written in the answer sheet. In your answers the solution to the problem should be clearly described.

Please note that some of the problem related figures are not exactly of the same size as given in the conditions of the problems. Therefore, when drawing the conclusion about the length of the line segment or other quantities, do not rely on the size of the figures. Pay attention to the conditions of the problems only.

Maximum score of the test is 59.

You are given 3 hours and 40 minutes for the test.

We wish you success!

Problem 1 **1 point**

Which number listed below belongs to the interval $(0,7;\ 0,8)$?

a) $\frac{3}{5}$ b) $\frac{7}{9}$ c) $\frac{6}{7}$ d) $\frac{8}{9}$

Problem 2 **1 point**

What is the greatest common divisor of the numbers $2 \cdot 3^3 \cdot 5$ and $3^2 \cdot 5^3$?

a) 6 b) 15 c) 30 d) 45

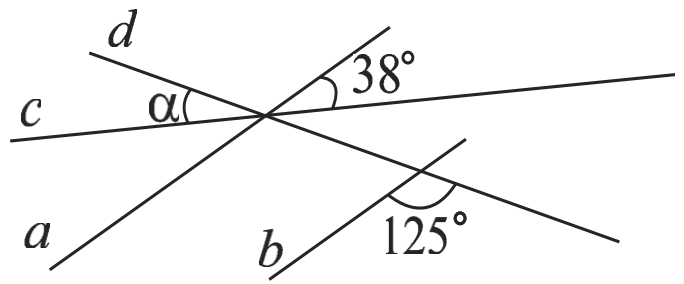
Problem 3 **1 point**

Coffee loses 12% of its weight when roasted. How many kilograms of unroasted coffee should we take in order to get 220 grams of roasted coffee?

a) $0,25$ b) $0,2464$ c) $0,3$ d) $0,34$

Problem 4 **1 point**

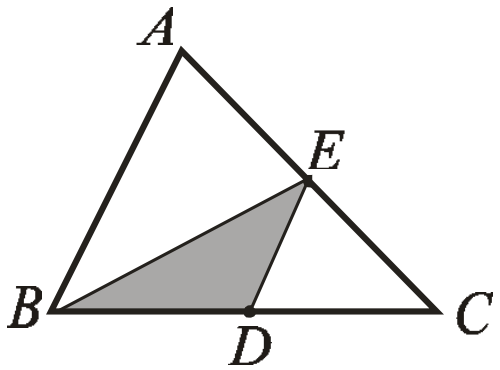
Straight lines a, b, c and d lie in one plane. The values of the two angles formed by these lines are shown on the figure. What is the value of the angle α, if the lines a and b are parallel?



a) 17° b) 24° c) 30° d) 38°

Problem 5 **1 point**

The triangle ABC is given in the figure. D and E are midpoints of the sides BC and AC, respectively. Find the area of the triangle BDE if the area of the triangle ABE is $36\,\text{cm}^2$.



a) $12\ \text{cm}^2$ b) $16\ \text{cm}^2$ c) $15\ \text{cm}^2$ d) $18\ \text{cm}^2$

Problem 6 **1 point**

The box contains two black and three white balls. What is the probability that if 3 balls are taken simultaneously and randomly out of the box, there is no white ball among them?

a) 0 b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$

Problem 7 **1 point**

Find the least positive integer n such that $8^{4n} > 2^{79}$.

a) 7 b) 8 c) 9 d) 10

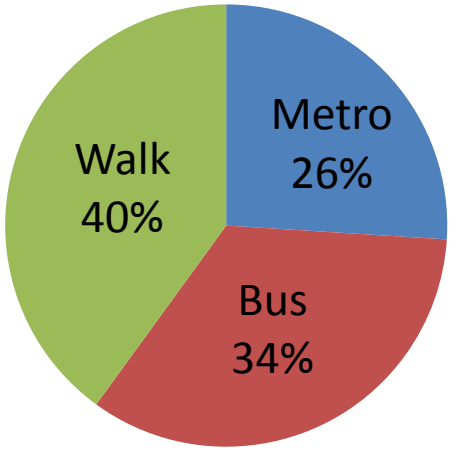
Problem 8 **1 point**

A car moves at a constant speed from Tbilisi to Kutaisi. By eight o'clock in the morning the car covered $\frac{1}{6}$ part of the planned route, and by 11 o'clock in the morning of the same day $-\frac{8}{9}$ part. What part of the planned route did the car cover by 10 o'clock and 30 minutes in the morning of the same day?

a) $\frac{65}{108}$ b) $\frac{57}{108}$ c) $\frac{8}{18}$ d) $\frac{83}{108}$

Problem 9 **1 point**

The students of one school were asked how they get to school. The results are displayed on the pie chart. Find the sector angle for walkers to the school.

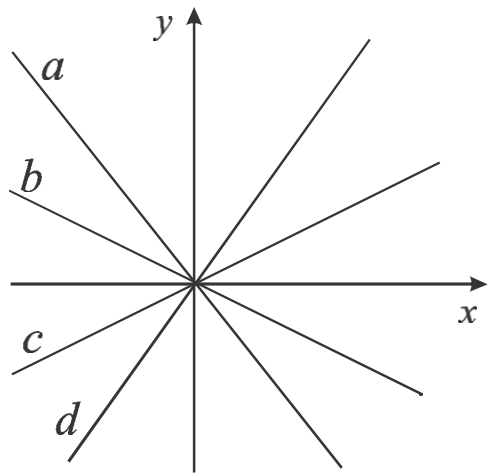


a) 120° b) 144° c) 154° d) 145°

Problem 10 **1 point**

The figure shows graphs of the following four functions in a rectangular coordinate system:

- $f(x) = 5x;$
- $g(x) = -4x;$
- $h(x) = -2x;$
- $p(x) = 2x.$



Which straight line a, b, c or d is a graph of the function $h(x) = -2x$?

a) a b) b c) c d) d

Problem 11 **1 point**

Which function listed below has a graph symmetric to the graph of the function $y=3(x-1)^2+2$ across the x-axis.

a) $y=3(x+1)^2-2$

b) $y=-3(x-1)^2-2$

c) $y=-3(x-1)^2+2$

d) $y=3(x+1)^2+2$

Problem 12 **1 point**

The fifth term of an arithmetic progression of positive numbers is three times as much as the second term. What is the ratio of the seventh term to the third term of the progression?

a) 3 b) $3,5$ c) 4 d) $2,6$

Problem 13 **1 point**

Let A be the set of all positive integers which, when divided by 4, give a remainder 1 and B be the set of all positive integers which, when divided by 4, give a remainder 3. Then $A \bigcup B$

a) is the set of all positive integers;

b) is the set of all positive integers which are multiples of 4;

c) is the set of all positive odd integers;

d) is the set of all positive integers which, when divided by 4, give a remainder 2.

Problem 14 **1 point**

Compute the scalar product of vectors $\left(\vec{a}-2\vec{b}\right)$ and $\vec{b}$, if $\vec{a}=(-1;\,0)$ and $\vec{b}=(-5;\,1)$.

a) 18 b) –26 c) 36 d) –47

Problem 15 **1 point**

$$\sqrt{(1-2\cos 45°)^2}-\sqrt{(1+2\sin 45°)^2}=$$

a) -2 b) $-2\sqrt{2}$ c) $2\sqrt{2}-2$ d) 0

Problem 16 **1 point**

How many faces has a prism, if the sum of the numbers of its all vertices and all edges is equal to 225?

a) 41 b) 45 c) 47 d) 52

Problem 17 **1 point**

Solve the following inequality $0{,}5^{x-3} < 10$.

a) $(-\infty;\ 2-\log_2 5)$ b) $(2-\log_2 5;\ \infty)$ c) $(-\infty;\ 3+\log_2 5)$ d) $(3-\log_2 5;\ \infty)$

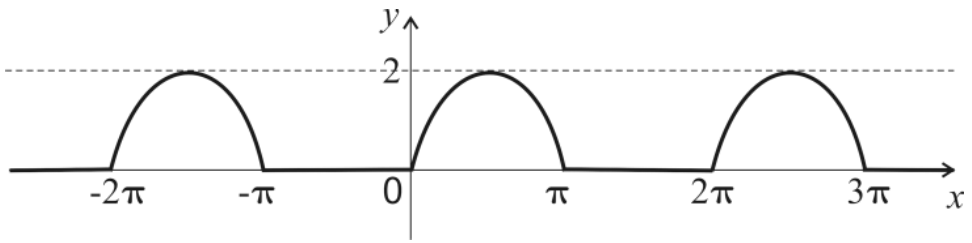
Problem 18 **1 point**

Find the mean of data, consisting of three non-negative numbers x, x^2, 1, if their median is $\frac{1}{4}$.

a) $\frac{21}{16}$ b) $\frac{7}{16}$ c) $\frac{7}{4}$ d) $\frac{7}{12}$

Problem 19 **1 point**

The graph of a function listed below is displayed in the picture. Find this function.



a) $y = 2\cos x\,|\sin x|$

b) $y = \cos x + |\sin x|$

c) $y = 2\sin x + |\sin x|$

d) $y = \sin x + |\sin x|$

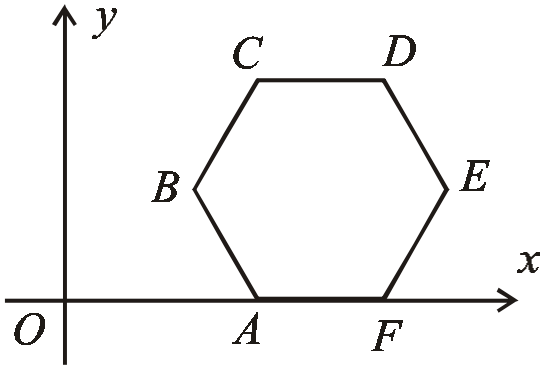
Problem 20 **1 point**

Find the radius of the circle circumscribed about the triangle ABC, if $AB = 6$, $AC = 9$, and $\angle A = 120°$.

a) $\sqrt{21}$ b) $\frac{\sqrt{63}}{\sqrt{2}}$ c) $\sqrt{57}$ d) $\frac{21\sqrt{3}}{\sqrt{2}}$

Problem 21 **1 point**

The regular hexagon $ABCDEF$ is given in the rectangular coordinate system Oxy so that the side AF lies on the x-axis (see the figure). Find the coordinates of the vertex E, if $OA = 6$ and $AB = 4$.



a) $\left(12;\ 2\sqrt{3}\right)$ b) $\left(11{,}5;\ \sqrt{3}\right)$ c) $\left(12;\ 3\right)$ d) $\left(11{,}5;\ \frac{3\sqrt{3}}{2}\right)$

Problem 22 **1 point**

Find the sum of the parameters b and c, if $x^2+bx+c=0$ and $3x^2+2x-5=0$ are equivalent equations.

a) -3 b) 1 c) -1 d) $\frac{7}{3}$

Problem 23 **1 point**

Find the value of the parameter b for which the lines defined by the equations $3x+2y-8=0$ and $2x-by=2y-5$ are perpendicular in the rectangular coordinate system Oxy.

a) $-\frac{10}{3}$ b) $-\frac{4}{3}$ c) 1 d) 3

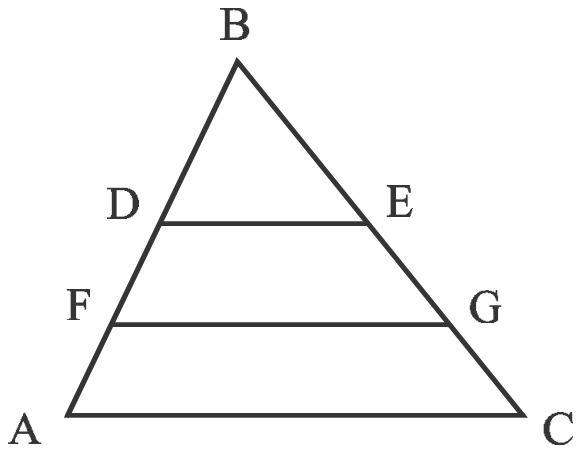
Problem 24 **1 point**

The length of the first circle's arc with the measure $60°$ equals the length of the second circle's arc with the measure $45°$. Find the ratio of the area of the first circle to the area of the second circle.

a) $\frac{3}{4}$ b) $\frac{9}{16}$ c) $\frac{4}{3}$ d) 2

Problem 25 **1 point**

Points D and F are taken on the side AB and points E and G are taken on the side BC of the triangle ABC so that the line segments DE and FG are parallel to the side AC and they divide the triangle ABC into three figures that are all equal in area (see the picture). Find the quotient of the length of the line segment FD to the length of the line segment AB.



a) $\frac{\sqrt{3}-1}{\sqrt{2}}$ b) $\frac{\sqrt{2}}{\sqrt{3}+1}$ c) $\frac{\sqrt{3}}{\sqrt{2}+1}$ d) $\frac{\sqrt{2}-1}{\sqrt{3}}$

Problem 26 **1 point**

Find the set of all the values of the parameter a, for which the function defined by the formula $y = \log_{a^2+1,5a} x$ will be an increasing function.

a) $\left(\frac{1}{2}; +\infty\right)$

b) $(1; +\infty)$

c) $(-\infty; 0)$

d) $(-\infty; -2) \cup \left(\frac{1}{2}; +\infty\right)$

Problem 27 **1 point**

The equation of a line given in the coordinate system is $y = 5x - 11$. Find the equation of the line which is obtained from the given line by the homothety centered at the origin with ratio 3.

a) $y = 5x - 33$ b) $y = 5x + 33$ c) $y = 15x - 33$ d) $y = \frac{5}{3}x - \frac{11}{3}$

Problem 28 **1 point**

The sum of the first 20 members of the increasing geometric progression is 50 times as much as the sum of the first 10 members of this progression. What is the common ratio of the geometric progression?

a) $\sqrt{5}$ b) $\log_2 5$ c) $\sqrt[5]{7}$ d) $2{,}5$

Problem 29 **1 point**

Find greatest value of the function $y = \dfrac{1}{2x^2 - 5x + 7}$.

a) $0,5$ b) $\dfrac{7}{3}$ c) $\dfrac{8}{31}$ d) 2,3

Problem 30 **1 point**

The base side length of a regular triangular pyramid is 4, and the length of the lateral edge is 5. Find the lateral surface area of the pyramid.

a) 6

b) $3\sqrt{21}$

c) $6\sqrt{21}$

d) $12\sqrt{21}$

Problem 31 **2 points**

Solve the system of equations

$$\begin{cases} \frac{3}{2}x + 2y = 7 \\ 2x - 3y = 5 \end{cases}$$

Problem 32 **2 points**

Two business partners have divided the profit in the amount of 80500 GEL in the ratio of 2:5. How much money did each one get?

Problem 33 **2 points**

Find the area of a triangle ABC, if $\angle A = 30°, \angle C = 45°$, and the length of the altitude BD drawn on to the side AC is 2.

Problem 34 **2 points**

Find the coordinates of the point on the line defined by the equation $5x+2y=4$, if it is known that the ratio of the abscissa and ordinate of this point is two.

Problem 35 **3 points**

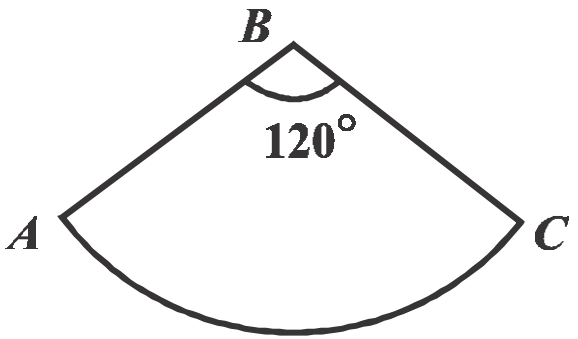
The total number of white and black balls in a box is 42. If one ball is drawn out of the box at random, what is the probability that this ball is white if it is known that adding 6 new white balls in the box will cause the increase of this probability by $\frac{5}{4}$-times?

Problem 36 **3 points**

Find $f(5)$, if the graph of the quadratic function $f(x)=ax^2+bx+c$ intersects the ordinate axis at point $(0;3)$ and its vertex is at point $(2;0)$.

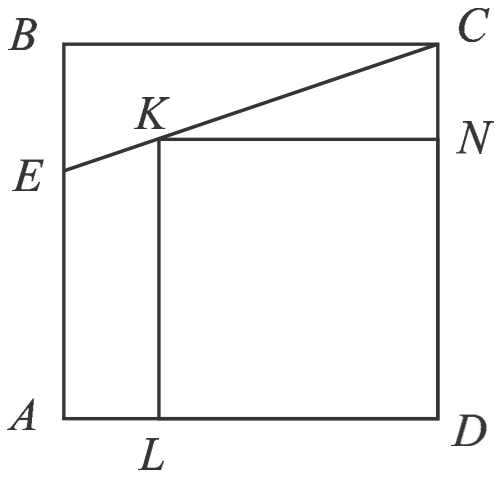
Problem 37 **3 points**

The net of the lateral surface of a cone is a circular sector with the central angle equal to 120°(see the picture). Find the base radius of the cone, if the height of the cone is $8\sqrt{2}$.



Problem 38 **4 points**

On the side AB of the square $ABCD$ a point E is taken so that $AE:EB=2:1$. A quadrilateral $LKND$ is a square, vertices L and N of which lie on the sides AD and CD, respectively, and the vertex K lies on the segment EC (see the picture). Find the side of a square $LKND$, if the side of the square $ABCD$ equals a.



Problem 39 **4 points**

Several workers did their work in 14 days. If there had been 4 workers more and a working day – 1 hour longer, the same job would have been completed within 10 days. If there had been 10 workers more and the working day – 2 hour longer, the same job would have been completed within 7 days. How many workers worked and what was the duration of the working day in hours, if it is known that labor productivity of all workers is the same?

Problem 40 **4 points**

Each straight line with negative slope which passes through the point $(3;\ 7)$ in the rectangular coordinate system Oxy, together with the x and y coordinate axes encloses right triangle. Find the smallest one among the areas of such triangles.